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The Effect of Uncertainty and Variability on the Economic Appraisal of the Nura Clean up Project in Central Kazakhstan

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SUMMARY

The River Nura is typical internal steppe river of Kazakhstan. It is concluded by a system of closed lakes of the Tengiz-Kurgaldzhinskaya depression which includes wetlands, an important site for wildlife and internationally significant one for ornithology. The river is subjected to natural cyclic process of variation of its annual flow causing variation of both Lake Tengiz salinity and the area of wetland. In the past, large amounts of mercury were discharged into the River Nura at Temirtau resulting in contamination and prohibited use of the river.

The mercury pollution seems not to be as serious as was first thought. So there is the possibility that the river can be cleaned up to the extent that its water could be used for the water needs of the new capital of Kazakhstan, Astana. This could have an adverse effect on Kurgal'dzhino due to the withdrawal of a part of the river flow and consequences of successive dry years.

Several Cost Benefit Analyses suggest that the competing water needs can be adequately met. However, they use a Certainty Equivalence Principle which may be appropriate for river systems in Western Europe, but not those in steppe Central Asia. By averaging over years, they neglect variability in natural water flow in the river, and consequently the resilience of the non-convex ecosystem. Also, the effectiveness of cleaning the river and the costs of alternative water supplies for Astana are uncertain. However, eventually, learning would lead to these uncertainties being resolved.

It is not appropriate to use a certainty equivalent analysis for this type of project, and we consider uncertainty and learning explicitly. We consider a strategy that is 'Robust' in the presence of many uncertainties, enabling sustainable use of the river, and recognizing that decisions may have to be made before some uncertainties are resolved.

1. Introduction

This paper considers some of the economic problems that can arise from Cost Benefit Analysis, as would be used for river and water projects in Western Europe, when applied to rivers and ecological systems with quite different characteristics. The specific environmental problem we consider is that of the Nura Clean Up Project and its impact on the Kurgal'dzhino Wetlands, which is the termination of the River Nura in Central Kazakhstan. The increasing use of such techniques arises both from the required methodology of international agencies, and from initiatives such as the Water Framework Directive of the European Commission, which often advocate the use of a standard appraisal technique across widely differing geographic areas. The problem is that assumptions and convenient short cuts may no longer be valid. The aim of this paper is to point out some of these problems, and to comment on the appropriate methodology for the economic analysis for such a river and wetland system. Although very simply modelled, we put the hydrological nature of the river and the ecology of the lakes as a central feature, as opposed to taking a standard appraisal methodology and then fitting the river and wetlands into it.

The River Nura flows into the closed lakes of the Tengiz depression which includes the Kurgal'dzhino wetlands, an internationally important site for wildlife. The wetlands are a closed system and mainly dependent on water inflow from the river Nura. It is recognised as very important wetland, being an essential habitat for many species such as Pink Flamingo, and White Crested Duck. There are now large international expenditures on wetland conservation and biodiversity through a GEF/UNDP project and NABU, mainly in order to restore its Ramsar status.[1] Increasing salinity and retreat of the wetlands, caused by extraction of water from the Nura, had been reversed when the discovery of substantial mercury contamination from past industrial activity km upstream from the wetlands at Temirtau caused the prohibition of Nura water use. Similar impacts from water level reduction and salination have been reported in past for Kurgal'dzhino as have occurred for the Aral Sea and Lake Balkhash. So proposals to extract and use water from the Nura, once the mercury contamination has been removed, could lead to similar problems. Stratiyenko (2004) suggests that an average phase of watering for the lakes of Korgalzhyn reserve is possible, without any regulating facilities, if the flows of the Nura and Kulanotpes[2] rivers are close to their natural average longstanding flow.

Calculating a safe amount of water extraction from the river before damage occurs is complicated by the variability of flow of the river. This variability is in fact required for the ecology of the wetlands. But if a constant amount of water were extracted, based on mean flow, then in some years flows into the Kurgal'dzhino wetland could be sufficiently low as to cause the saline Lake Tengiz to dry up. Years of mean, or higher, flow occur only one year in three. So, an important part of any study of the river should be to find out how variability impacts on the hydrological and ecological cycles. These should have some presence in an economic resource model, albeit in a simplified way.

Several Cost Benefit Analysis and other economic studies of the Mercury Clean Up Project have been undertaken[3]. These suggest that the competing water needs of the new city of Astana and the Kurgal'dzhino reserve can be adequately met[4]. However, they use a Certainty Equivalence principle such as would be commonly used for river systems in Western Europe, where mean, or expected, values are used instead of random variables. We look here at the extent to which the certainty equivalent methods implicit in standard Cost Benefit Techniques will be applicable when applied to a very different river system with a different hydrological regime. Of course, there are very good reasons for taking a standard appraisal framework. One important one is that the treatment of uncertainty by replacing uncertain quantities by their expected values, and then proceeding as though these variables were certain ones, has many computational and analytical advantages. We see this throughout our analysis where

[1] See Bragina et al. (2001)
[2] This is the other main river entering into the wetlands.
[3] See World Bank (2003), Jacobs Gibb (2004)
[4] Discussion of the Nura in the context of Water in Central Kazakhstan in general can be found in Tanton et al. (2000)

we are only able to consider slight and simple modifications and extensions to the standard analysis. However, our aim is to show that treating uncertainty and variability explicitly can make a big difference to the conclusions reached and thereby the policies implemented.

We take as a starting point conclusions obtained for a World Bank study for the Nura Clean Up project. This project has the following intentions.

> "*The project objective is to provide users in the Nura-Ishim basin with access to safe, reliable, and affordable water supplies by cleaning up the mercury pollution, instituting effective water quality protection measures, and optimizing use of available water resources. Improved water supply from the Nura will in turn help meet additional demand for Astana; increase reliance on local water resources by basin users; and increase summertime flows to the Kurgal'dzhino wetlands*."[5]

It is not all together clear that all of these objectives can be met. For example, contrast the above with "*The only threat to ecosystems of the property, Tengiz-Korgalzhin Lakes in particular, may be changes of the water regime. ...*

> *Until now, the water main inflow to the Tengiz-Korgalzhin Lake system, the Nura River, was not used extensively for irrigation purposes due to its high contamination. Plans to clean up the river should (ought) therefore not lead to higher use of water resources out of the system. The very shallow Lake Tengiz could dry out rapidly in this case."* (Bagarina et al.(2001))

However, because of the way that variability of water flow is treated, quantity of flow into the Kurgal'dzhino wetlands often does not emerge as a problem or constraint. This is because the level of extraction is thought to allow the required flow into the wetlands to be met on average. But, variable flow of the river, and sequences of dry years may make ensuring appropriate flow into Kurgal'dzhino problematic. The World Bank Study reports that 90% of projected Astana water demand could possibly be met from diverting water from the River Nura. This represents 90 m. m^3 per annum. In the next section, we shall see that water extraction on this scale would have immense consequences for the fresh water/ saline balance of the terminal complex of the Nura. Unless a strong system of water management is put in place, then given the costs of the various options and the fact that this water is likely to be much cheaper than any of the other options, presumably this level of extraction is what will happen. If the planning methodology treats the wetlands as a residual, then the potential damage to the wetlands should be included in costs. We consider the possible implications of doing this given the uncertainties and variability involved.

We proceed in the following way. The next section looks briefly at the River Nura and the Kurgaldzhino wetlands and the variability of flow. We contrast the pattern of flow and its distribution to some of those found in western Europe. We do not seek to provide a full analysis, but to show that a skewed distribution such as the log-normal captures the overall pattern much more closely than a normal distribution would, which is the distribution implicit in the certainty equivalent approach. The log-normal distribution can arise out of a Wiener stochastic process and it is possible to use results on when such a process will reach a particular boundary. This will be important if the wetland system has threshold values and we then see that such thresholds are likely to be present. These come from how water flow might impact on the wetland and lake system. Recent results from the economics of non-convex eco-systems, such as that of shallow lakes, suggests that both stock effects and thresholds are likely to be important in how the ecosystem of the wetland operates. So we then survey what

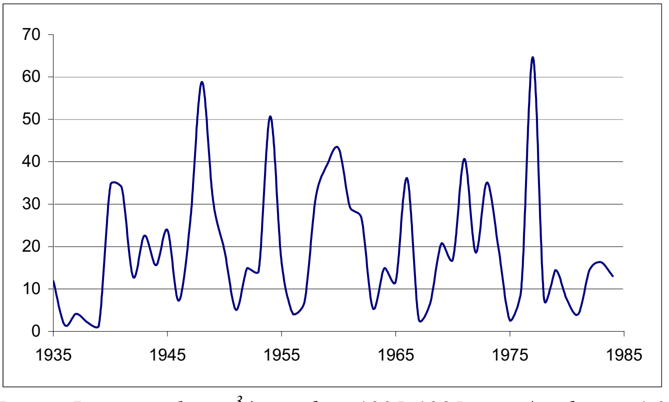
[5] page 66 of World Bank (2003)

the presence of these catastrophic effects might imply, and the possibility that these threshold levels will be reached as a consequence of variability.

Then we look at what can result from considering alternative distributions to that of the Normal, and alternative functions to that of the quadratic, which are both required to fully justify the certainty equivalent approach. We provide some simple explicit examples. They are simple because of the difficulties in replacing uncertain variables by their expected values, which is why this procedure is so often used. We look at the effect of alternative uncertainty specifications within a two period model, where stock effects are present, and we look at how the effects of skewed river flow distributions will impact on irreversibilities in the water extraction decision. Finally, we look briefly at what robust policies might be for this type of problem.

2. The River Nura and The Kurgaldzhino Wetlands

The River Nura is a typical steppe river, with 85 % of flow occurring during the spring floods. Surface water flows in Central Kazakhstan show considerable non-uniformity. Average data on the annual flow of the River Nura shows that the spring flow exceeds the annual average value by 7-8 times. In the autumn, the monthly average flow is an order less than the annual average value, and in winter it is close to zero. The maximum flood can be 4-5 times greater than the average. In dry years, the floods can be ten or more times smaller than average, reducing the annual average flow of the river by an order of magnitude. At the end of the 1930s there was a period of 5 floodless years in succession on the Nura, when the total of five years' flow was less than one average year. Annual flows for the Nura are shown below:



Nura at Romanovskoe m^3/sec.: data 1935-1985 mean/median = 1.32

All rivers have very variable flow both within years and across years. However, river flows in Western Europe are very different from those in Central Asia. The first substantial difference between Central Asian and Western European Rivers is that of the pattern over the year. The following table shows the monthly pattern of flows for the River Nura and for the River Great Ouse, as just one example, in England. Data for the Nura shows a spiked pattern whereas that for the Ouse it has a ‘U’ shape.

Monthly and Annual Average for flow (m^3/sec.) over a 50 year period
River Nura at Sergiopolskoe, Romanovskoe, and on R Ouse (England)

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Annual
Sergiopolskoe	0.06	0.04	4.80	48.40	7.90	2.96	3.01	1.45	0.88	0.86	0.66	0.23	5.94
Romanovskoe	0.48	0.40	0.46	159	45.5	11.0	5.87	3.42	2.73	2.89	1.79	0.69	19.6
R Ouse	29.3	26.3	21.0	15.8	10.1	7.69	5.90	5.11	6.13	6.23	7.89	22.6	16.4

Another important difference between the Nura and Western European rivers is that the latter usually show serial independence of flows, whereas, for the Nura, there is a pattern of wet and dry years.

Cycles of High and Low Flow States of the River Nura between 1935 and 1983

Period	No of Years	Flow State
1935 – 1940	6	Low
1941 – 1949	9	High
1950 – 1957	8	Low
1958 – 1962	5	High
1963 – 1969	7	Low
1970 – 1973	4	High
1974 – 1983	10	Low

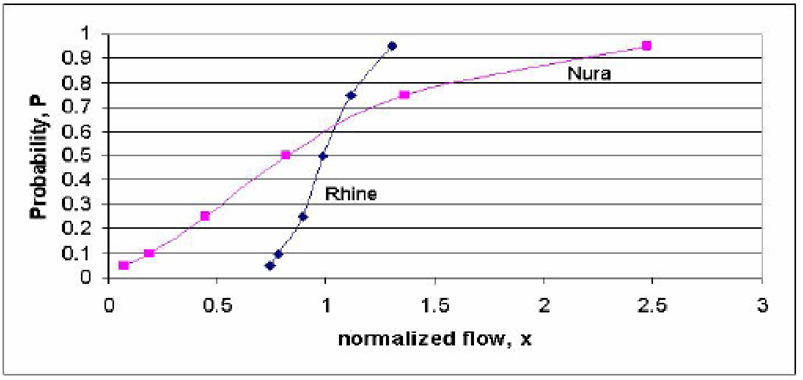
For most rivers in western Europe, the ratio of maximum to minimum annual flow is between 2 and 5, and the mean and median flow are very similar. For the Nura, the distribution of annual flow is shown in the following table. There is a substantial difference between the mean and median flow, indicating a highly skewed distribution very different from the distribution for many western European rivers.

Probability Distribution for Flow (million m^3 / year) on River Nura
X such that Prob (x > X) = P

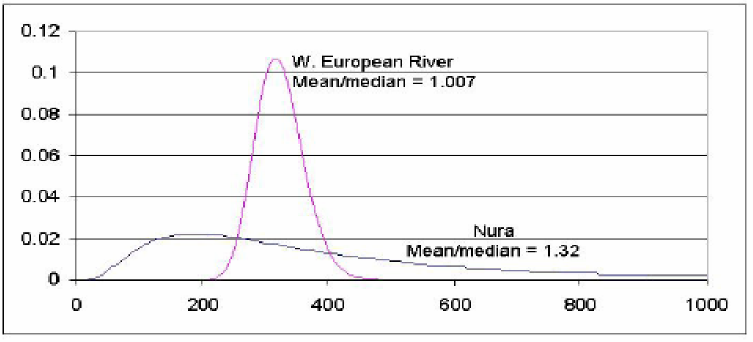
	Mean	Ratio of	Probability P%					
		Mean/Median	5%	25%	50%	75%	90%	95%
Sergiopolskoe	188.0	1.30	492.0	217.6	145.1	69.4	34.7	18.9
Romanovskoe	619.1	1.22	1535.8	845.2	507.7	274.4	116.7	47.3

From this table, an approximate distribution for annual flows can be obtained. This is shown in comparison to the distribution for the Rhine in the figure below where both series have been standardised to have the same mean of 1 for comparison.

By looking at the density rather than the cumulative distribution function, it is also clear that the distribution for the Nura is highly skewed. Evidence from stochastic modelling for other rivers suggests that a log-normal distribution would be a good first approximation to this distribution. The log-normal is useful in that the 2 parameter version is described entirely by the mean and median. Using the formulae for average (mean) and 50% probability (median), an approximate log-normal distribution is shown in the following diagram for both the Nura, and a river where the mean/median ratio = 1.007, such as the Rhine.



Probability, P, that flow at most x



Log Normal distributions for Nura and a typical W. European River

The mean flow of the Nura has cumulative probability of 0.63, so that for 63% of years (almost 2 in every 3), flow is less than average. The most likely flow (or mode) is half of the mean value. So, the use of an expected or mean value is likely to be misleading as a representation of what will happen in the majority of years. The important feature to note here is that the range for the Rhine is very limited, and high and low extremes are much more likely for the Nura than they are for the Rhine.

The Nura basin has a high level of pollution, by heavy metals, oil products, and other chemicals. The main pollutants are mercury and its combinations. Mercury wastes have come to the river for many years with outflow from the chemical plant ("Karbid") in Temirtau as a result of accidents and old technological processes. Technogenic silts of the river are annually carried by flood waters downstream from Temirtau. It is likely that some has reached the lakes of Kurgald'zhino. However, the accumulation of mercury, other heavy metals and pollutants in the biomass and the environment of these lakes has not been surveyed. It appears that most of the mercury contamination is localized to the area where it originated, and that removal of the silt from this region would reduce risk from mercury substantially[6]. Heaven et al. (2000a) propose measures that would reduce mercury levels by 95% and indicate that this could be a cost effective solution to water shortage problems of the region. However, cost effectiveness needs to include environmental and ecological costs arising from water loss to the wetlands. We consider here only water quantity aspects, and not water quality. Considering both together would introduce substantial extra complexity.

Variability of Flow and the Kurgal'dzhino wetland

The River Nura is the main surface water flow into the terminal lakes of the Tengiz-Kurgaldzhino depression. Its delta forms wetlands of international significance. There are more than 60 fresh and saline lakes of different salt content, most situated in the delta. These wetlands and Lake Tengiz form a complex and closed saline/fresh water system. The dynamics of this are complex and not fully understood. Precipitation occurs mainly in spring, and in summer, as the maximum temperature is over 40ºC, substantial evaporation of about 0.7 m. per year takes place. The variability of water inflow from the River Nura and the diversion of its water has caused in the past considerable variation in their extent. The minimum for more than a century was observed on these lakes in 1929-1940, and in 1934 the depth of Lake Korgalzhyn was less than 0.6 m.

Stratiyenko (2004) reports that the optimal level of the Korgalzhyn lakes corresponds to a volume of at least 280 million m^3. At present, Lake Korgalzhyn is characterized by lower than average levels. The total area of the lakes amounts to about 133 km^2 with a volume of water about 200 million m^3. When the water level is high, the area will increase to 170 km^2, with a volume of more than 300 million m^3. In a maximum phase (1948-1949), the area of Lake Korgalzhyn waters reached 297 km^2, with a volume of 583 million m^3. At a recent minimum (1964-1965), the area decreased to 63 km^2 with a volume of 25 million m^3, the average depth being 1.6m.

The fresh water lakes have often an outflow. Water flows from the freshwater Lake Korgalzhyn into saline Lake Tengiz. This is the biggest lake of the steppe zone. Its extent varies considerably. At present, the average depth of Lake Tengiz is up to 2.2 m. with an area of 654 km^2, and a volume of water up to 816 million m^3, average depth up to 1.4m. At a minimum, the area 375 km^2, with a volume 150 million m^3, and average depth of 0.4m. When there are successive years of high water inflow, evaporation is a limiting factor for

[6] Details of the mercury contamination of the River Nura can be found in Heaven et al. (2000a, 2000b).

water levels of the Nura lakes in the 2nd-3rd year. A considerable part of the runoff will then run into Lake Korgalzhyn, and in the case of its being full – into Lake Tengiz as well. This causes changes in the salt content in the water which can vary in the range from 22 to 127g/litre.

The World Bank Study reports that 90% of projected Astana water demand could possibly be met from diverting water from the River Nura. This represents 90 m. m^3 per annum., or equivalently approximately 2.85 m^3/sec. It has been estimated that a reduction in flow in the River Nura of 1 m^3/sec. over a year degrades 40 km^2 of the Kurgal'dzhino wetland.[7] So that up to 114 km^2 of lake area could be lost from this level of extraction. The World Bank study reports that the cost of water in Astana coming from the Nura is \$0.07/m^3 compared to the next cheapest alternative of \$ 0.17/m^3. So, it is not impossible for there to be substantial losses in water flow ino the lakes in a a dry period, such as 1936 to 1939. Then, annual average flow was between 1.5 and 4.1 m^3 per sec., and consideration of the state of the wetlands would be a binding constraint on the amount of water that could be extracted.

Whilst the World Bank Nura Clean Up project includes consideration of the Kurgal'dzhino wetlands, there are many difficulties and problems in doing this even if there were more knowledge about the ecological behaviour of the area[8]. An important aspect is the extent to which the analysis incorporates uncertainty and variability, especially of sequences of dry years. The message that we take from this for consideration of variability is that there is *in extremis* a danger if water inflow were to be substantially reduced that Lake Tengiz could dry up completely. This would lead to the Korgalzhyn lakes becoming saline with a consequent loss of its ecological value. Even if such a catastrophe were avoided, reductions in water inflow will lead to a reduction in the area of the lake with consequent loss of its special habitats. As water inflow is reduced into the lakes, there will be an increasing environmental cost due to two factors. One is the value of area of lost habitat, the other is a cost to being nearer to the potential of a salination catastrophe for the lakes, either in the near or long term. We capture this, in the worked examples of sections 4 and 5, provided to back up our arguments, by means of a quadratic cost function. This is for ease of manipulation so that the extra complications arising from including an uncertain water flow are manageable. The next section looks at the implications of potential catastrophe.

3 Threshold and Catastrophe Effects arising from the Wetlands and their Consequences

[7] Personal Communication from Prof. T. Tanton

[8] Volume 4 of Jacobs Gibb (2004) p.5-8 says that " However, given its importance, it is surprising that almost no research has ever been carried out on the area from any scientific perspective: biology, ecology, hydrology, etc. Specific to this project, there are no records of water levels in Tengiz Lake so it has been impossible to establish a relation between inflow from the Nura and water levels either in Tengiz or the wetlands upstream. Two studies have attempted to estimate the water requirements of the area. These arrived at estimates of 150 million m^3 per year and of 400 million m^3 per year."

The wetlands are a complex system, and have the important characteristic of being shallow lakes. So, for subsequent purposes, we use some results from recent work on such lake systems. These point to the problems that will occur if the particular characteristics of these are neglected in economic models. Maler, Xepapadeas and de Zeeuw (2003) provide a model for a shallow lake which is subject to eutrophication due to high nutrient intake, especially phosphorus. This basic model applies as a metaphor for the general ecological problems of lakes. The basic phenomenon is one of hysteresis, where two separate states are possible. One is a high biodiversity state and the other a low one. The system has a possibility to flip from one state to the other. Such a phenomenon has been observed for the situation of dessication and increasing salinity of closed lake systems - the Aral Sea being a well known example.

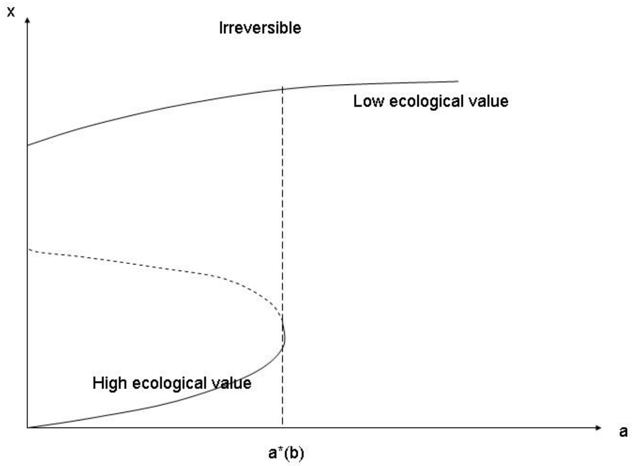
Maler et al. use a differential equation for the stock effect arising from a pollutant x(t) being determined by an input of the pollutant y(t) being given by

$$\dot{x}(t) = a(t) - bx(t) + r\frac{x^2(t)}{(x^2(t)+1)}$$

In this case, we could think of *x(t)* as being area of degraded wetland, or the effect of increased salinity, and *a(t)* as being water loss through Nura water diverted elsewhere.

The behaviour of the solution to this differential equation system can be seen by considering the solution for *x* for a constant *y* depending on the value of the parameter *b*. For low values of *b*, increases in *a* will lead to increases in *x*, but the changes are fully reversible. As *b* increases, then for a range of values of *b* there is the possibility of hysteresis where there are two stable equilibria, and as *a* increases the system can flip from one state to the other. However, it is not always possible to move back to the initial state just by reducing the value of *a*. The value of *a* has to be reduced by an extra amount in order to attain the original state. So whilst it is possible to get back to the original state, there is a cost.

For values of *b* in the upper part of the range, the system is trapped in the low ecological value state should *a* ever exceed a critical value $a^*(b)$, and only a change in the value of *b* can restore the system to its initial high ecological value state.



So there will be a value, b_1, below which changes in *a(t)* are fully reversible, and a value b_2 above which reversibility is not possible by changing *a(t)* alone, and in between, when

$b_1 \leq b \leq b_2$, reversibility is available at a cost of reducing *a(t)* below the level at which the system originally flipped. The non convexity here arises from the cusp-catastrophe nature of the solution to the differential equation system. In a convex system, only one solution would arise and the consequences of actions are fully reversible.

Non-convexities imply ecological thresholds, so that if large damage were to be imposed then the consequence would be irreversible. But in the case of the shallow lake, large damage can arise from a small increase in water abstraction, *a*, and when *a* is a stochastic process such a possibility may arise inadvertently. If the situation is reversible, then whilst there will be a cost we could incorporate this into the decision problem or Cost Benefit Analysis. When we end up in the irreversible case, separate account may need to be taken of this.

Threshold and Catastrophic Effects.

The previous discussion showed how thresholds can arise from the differential equations describing the ecology of a lake system. Such catastrophic effects could arise either from the dynamics of the lake system as discussed earlier, or from irreversible salination due to reduction in water inflow because of diversion of Nura River water to other uses and/or a succession of dry years. Catastrophe arises from the crossing of a threshold, the location of which we are uncertain about. In this section, we discuss how incorporating catastrophic consequences may influence the optimal decision, and the implications of irreversibilities.

Catastrophes are those events which have large impacts but may occur only with small probability. How the risk of catastrophic events affects current policy towards a stock variable,such as water level in the wetlands, or cumulative water withdrawal, depends crucially on a number of features of how catastrophes are modelled: (i) what triggers catastrophic events; (ii) whether the risk of catastrophic events is avoidable; (iii) what the impact of a catastrophe is, in particular whether the costs of a catastrophe are reversible or irreversible.

The first economic analysis to consider such effects was that of Cropper (1976). It assumed that the stock accumulates in a stochastic fashion, but it is known for sure that once the stock level reaches a certain critical threshold, then a catastrophic event occurs, which causes a temporary, fixed, reduction in well-being. Above this critical level of water in the system, the stock has no effect on well-being. However, the loss of well-being caused by the catastrophe is reversible, since the effect disappears once the stock rises above the critical level. Cropper shows that compared to no catastrophe, policy would require a reduction in extraction, to reduce the likelihood of triggering such losses.

Clarke and Reed (1994)) consider a situation, in this application to water levels in the wetland/lake complex, of when there is no catastrophe, well-being depends negatively on the reduction in the water stock. There is a probability (hazard) of a catastrophe occurring, and if it does occur well-being falls sharply to some base level, which does not depend on the stock level, and remains at this level for ever. Well-being is the ecological value of the wetlands. In this sense, the catastrophe is *irreversible* in its effects. The probability (hazard rate) of a catastrophe occurring at any particular date is known, and may depend on the stock. They show that there are two effects at work. If a catastrophe occurs, the stock no longer affects well-being. This *reduces* the expected marginal damage cost of current water extractions. On the other hand, an increase in extraction may increase the probability of a catastrophe occurring. This increases the expected marginal damage cost of current extractions. Thus, if the hazard rate is either independent of the stock, or varies very little with the stock, the first effect will dominate, and the optimal policy involves a *higher* level of extractions than if there was no risk of catastrophe. If the marginal effect of current extractions on the risk of a catastrophe is very high, so risks are easily avoidable, then the second effect dominates and optimal current policy should be to reduce extractions. In between the effect is ambiguous.

Tsur and Zemel (1998) have the same structure of risks of catastrophe, but they now allow for the effects of a catastrophe to be reversible, in the sense that a catastrophe just involves incurring a once-for-all penalty in terms of a loss of ecological value of the wetlands. Moreover, if catastrophes are reversible they can also be recurrent. They show that reversible catastrophes, whether recurrent or not, always lead to a lower level of extractions than would be the case without catastrophes. Tsur and Zemel (1996) consider what happens when uncertainty is endogenous in the sense that a catastrophe is triggered by the level of the stock, but it is not known what that stock is. Learning takes place, in the sense that if no catastrophe has occurred up to now, then the critical stock level must be at least as low as any past stock level. A catastrophe has the same kind of reversible effect as in Tsur and Zemel (1998) i.e. society incurs a once-for-all penalty if a catastrophe occurs. They show that the optimal policy can be characterised in terms of an interval of steady-state stock levels. The upper end of this interval for the stock of water is the steady-state stock that would be set in the absence of catastrophes. If the initial stock of water is above this level, then society just aims for that steady-state and ignores catastrophes. The reason is simple - society knows that catastrophes cannot occur in the relevant range of stocks. On the other hand, if the initial stock is below the lower end of the interval of steady-state stocks, society should move towards that lower end, but more cautiously than if there were no catastrophes. The interesting case is where the initial stock of pollution lies in the interior range of the steady-state stock levels. In this case, society should immediately leave the stock level as it is. The analogy might be that because any action might trigger catastrophe, but that has not so far happened then we should now stay where we are because the risk that the next step will trigger one is now very high.

In terms of water withdrawal, if water levels exceed that at which catastrophic effects could occur then we are able to use water up to the point where a risk of catastrophe emerges, but cautiously because of the presence of catastrophic effects. However, if we think we are in the intermediate range where catastrophic effect is possible, then we should maintain the stock of water at its current level. Of course, in this case the stock is varying exogenously. So we need to consider the whole path for water level over the relevant future, rather than just a single level. We look at a simple analysis of this and how it enters into certainty equivalent studies in the next section.

In summary, the impact of the risk of uncertainty on optimal water policy shows there are no unambiguous results. Depending on the features of catastrophes set out at the beginning of this section, compared to policy with no catastrophes the risk of catastrophe could call for weaker action now, no further action, somewhat tougher action, or drastic action to stop any further effects on water levels. However, the balance of results would suggest that the possibility of catastrophic effects should lead to a reduction in planned water extraction, possibly a very drastic reduction.

Catastrophe and A Stochastic Process Interpretation Of Log-Normal River Flow.

Under what conditions might a threshold for catastrophic effects be reached? Given the variability in flow, might this happen as part of the natural hydrological cycle? If it does then lowering the flow will increase the probability that the threshold would be reached, and we would expect this to occur sooner than otherwise. These are questions that can be answered using a stochastic process model for river flow. The Log-Normal distribution is used widely as a good representation of various data processes. It is especially useful in modelling various financial series such as stock market prices. As a consequence, it has been very widely studied, and there are many important results that can be used directly.

An important reason why the Log-Normal distribution may represent data so well is that it arises from a Geometric Brownian Motion representing stochastic proportional growth. For river flow, Lefebvre (2002) and Labib et al. (2000) found that diffusion processes can be a good representation of river flow data, albeit over rather short time periods. They find this process to be the most appropriate. It yields the Log-Normal distribution for river flows, and also replicates the pattern of serially correlated years of high flows and low flows, which is a feature of the Nura.

In this section, we shall consider some of the implications arising from this representation of the river. Of course, it is an approximation in that it is the distribution of annual flows that will give rise to a Log-Normal distribution, rather than the within year flows. *Geometric Brownian motion with drift* or the *standard diffusion process*, is a process given by:

$$dS = \alpha S dt + \sigma S dz$$

where dz is the differential of a standard Wiener process.

The properties of this model are set out in Dixit (1993) and its use in Financial Economics and Investment in Dixit and Pindyck (1994). There are undoubtedly many results that can be derived relevant to the problem of water and river management from these models. Here, we utilize just one set – 'first hitting times'. For a threshold that corresponds to a constant level then we can obtain results as to when this threshold will be reached by a stochastic process for water inflow. For the management of the wetlands, we would be interested in a lower boundary value that corresponds to irreversible damage in the wetlands. This is given by the expected first hitting time E(T*) which is the expected first time for a stochastic variable starting from a value of S_0 of hitting a barrier at level a with $0 < a < S_0$. This is given by :

$$\begin{aligned} E(T^*(S = a)) &= \frac{1}{\tfrac{1}{2}\sigma^2 - \alpha}\ln\left(\frac{S_0}{a}\right) \quad \textit{if } \alpha < \tfrac{1}{2}\sigma^2 \\ &= \infty \quad \textit{if } \alpha \geq \tfrac{1}{2}\sigma^2 \end{aligned}$$

The expected time falls as the variance term in the process increases in relation to the trend term. So, for a sufficiently high variance, there is a finite expected time at which the lower boundary will be reached even if there is a positive trend component. When the current water level in the wetlands is high, in relation to the threshold catastrophe level, S_0/a may be high causing there to be a long time before hitting the threshold through natural variability[9]. If S_0' is an average stock

[9] We are assuming here that the water stock in the wetlands will follow a process similar to that of water flow in the river.

of water in the wetlands at the point with an extraction/diversion strategy in place, and this is a proportion β of that with no extraction, then expected hitting time is increased by a factor of $\ln(1/\beta)/(\sigma^2/2 - \alpha)$.

For $\beta = 0.5$, $\sigma_N^2/2 = 0.46$ [10] and $\alpha = 0$, the expected time to reach a threshold increases by 31.5%. For a western European river such as the Rhine, where $\exp(\sigma_R^2/2) = 1.007$ and if for the Nura, $\exp(\sigma_N^2/2) = 1.3$, then $\sigma_N^2/\sigma_R^2 = 37.6$, hence the expected time that a low level of flow on the Nura will be reached is about 38 times sooner than that for a similar threshold but for a river with western European flow characteristics.

This is a qualitatively different result from that for a Normal distribution for water level, such as would be generated by an Arithmetic Brownian Motion.

In this case, $E(T^*(S = a)) = \dfrac{S_0 - a}{|\alpha|} \quad \textit{if} \quad \alpha < 0$; $E(T^*(S = a)) = \infty \textit{ if } \alpha \geq 0$

So that for no trend (or drift term) then the expected time to reach the threshold level, *a*, is infinite[11], independent on the distance between the starting point for the process, S_0, and the threshold level. This is a possible justification for the logic behind the assumption that because expected withdrawal of water leaves expected inflow into the wetlands above the expected threshold, then no problem exists for the ecological status of the wetlands.

If there is no positive drift component over time, then the level *a* is certain to be reached at some time, but the time to reach the threshold does not depend on a variance term.[12] In conclusion, whereas for normally distributed water flow and low variance we may be able to ignore the lower boundary on expected terms, for log-normally distributed flow with a high variance then it is essential that this boundary be taken account.

4. Uncertainty and Precaution in Optimal Water Policy

In order to provide a simple worked out example to back up the arguments that we make, we shall use a simple two period model relating to water flow in the river and water extraction. This allows for a stock based problem, such as those for the Nura where the cumulative and dynamic effects are important for the ecological damage caused. We look at how increasing

[10] Corresponding to mean/ median ration for the River Nura

[11] However, a different perspective is obtained if we look at the probability of hitting a barrier $0 < a < V_0$, which is

$$\pi = \exp\left(-\frac{2\alpha(S_0 - a)}{\sigma^2}\right) \text{ if } \alpha > 0 \text{ and } \pi = 1 \textit{ if } \alpha \leq 0$$

[12] Climate Change leading to a reduction in river flow would be one possible reason for a negative trend term.

variability in river flow changes the extent to which a precautionary approach should be adopted, and then contrast this with the case where a Certainty Equivalent approach is used.

The aim of the model is to calculate the amount of precautionary behaviour, and show how this depends on the degree of variability. An important limiting case is that in which precaution means that no water would be removed from the river at all. Even though it is a simple model, it becomes impossible to obtain explicit solutions except for special cases and functional forms. We therefore use specifications which allow us to reach specific conclusions. The aim is to demonstrate the possibility rather than the necessity of such results. The main difficulty arises in calculating the expected values of random variables when we depart from the standard case of quadratic objective functions and Normal probability distributions.

The condition of the wetlands is given by the amount of water contained within them. Damage to the wetlands occurs as a result of several years depletion of water, so we take this to occur at the end of the second period and to depend on how much water has been removed over the two periods. If there were no water withdrawals, the quantity of water would be a quantity $\overline{W_t} + x_1 + x_2$, where x_t is the stochastic component of inflow from the river in period t. However, if mercury is removed from the river, there is a base level of water extraction in period t, if no action is taken, given by an amount b_t . This extraction can be reduced, for example by policy interventions[13], such as water conservation measures, leakage reduction, or the use of alternative sources of supply.

Damage depends on the quantity of water extraction less additional flow from the river.

The level of water extraction that is consequently saved, for instance from alternative sources is denoted by a_t, so net extraction of water from the river in period t is $b_t - a_t$, and the total net stock of water extraction is $S_2 \equiv b_1 + b_2 - a_1 - a_2 - x_1 - x_2$.

The marginal benefit of using alternative water to that from the Nura is given by the linear expression

$MB(a_t) = \delta_0 + \delta_1 S_2 - \alpha_t g(x_t)$, t = 1,2

where x_t, the random variable representing stochastic additional water flow has a probability density function of *f.* The logic to this is that extra flow of water can replace water extracted, and so reduces the marginal benefit from using water from alternative sources. It separates marginal benefit into a certain and random component.

This marginal benefit corresponds to a total damage cost from water use of

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[13] Such as those set out in the World Bank(2003) study and the Jacobs Gibb (2004)[13] report. Volume 4 Appendix B sets out 8 possible options and gives costs for these..

$$D = \delta_0 S_2 + \delta_1 \frac{S_2^2}{2} + \alpha_1 a_1 g(x_1) + \alpha_2 a_2 g(x_2)$$

For simplicity we take a quadratic cost of using water from alternative sources, or of water conservation measures, so that the marginal costs of action a_t is $\gamma_t\, a_t$. The non quadratic part of the overall cost is restricted to that arising from the function g and this enables standard results on expected values to be used and explicit results obtained.

For future reference, the full planner's problem is:

$$\min_{a_1, a_2} E \left[\begin{array}{l} \dfrac{\gamma_1 a_1^2}{2} + \dfrac{\gamma_2 a_2^2}{2} + \delta_0 (b_1 + b_2 - a_1 - a_2 - x_1 - x_2) + \\ \dfrac{\delta_1 (b_1 + b_2 - a_1 - a_2 - x_1 - x_2)^2}{2} + \alpha_1 a_1 g(x_1) + \alpha_2 a_2 g(x_2) \end{array} \right]$$

This problem is to minimise total expected costs over the whole of the planning period, including damage to the wetlands and the cost of reducing this by alternative actions to that of taking water from the Nura.

In order to be able to obtain an explicit expression for expected value, we take *f* to be a Log-Normal distribution and we use a constant relative risk aversion form for g,

$g(x) = x^{1-\lambda}/(1 - \lambda)$ with $0 < \lambda < 1$, which ensures that $g > 0$. The parameter λ allows for variability to impact to a greater or lesser extent on the decision problem. For $\lambda = 0$, g will be a linear function and so only the mean of the random variable will be relevant to decisions. For $\lambda = 1$, g will be a logarithmic function, but $g > 0$ only for $x > 1$.

We solve this problem by backwards recursion. First, we obtain the optimal decision for period 2, a_2 , based on the stock of water in the wetlands. This depends on the decision made in period 1, a_1 , and from these we can obtain a value for the expected value of the overall objective function. For the period 1 decision, we know the costs of different values of a_1 and the overall expected value. This enables the optimal value for a_1 to be calculated recognising the consequences for period 2 decisions.

<u>Period 2.</u>

Define: $\widetilde{S}_2 \equiv (b_1 + b_2 - a_1 - x_1)$. This is the total effect of extraction on the stock of water in the wetlands at the start of period 2, after reduction in period 1 but before any decision is made about reduction in period 2.

Define: $V(\widetilde{S}_2) \equiv \min_{a_2} E\left[\dfrac{\gamma_2 a_2^2}{2} + \delta_0 (\widetilde{S}_2 - a_2 - x_2) + \delta_1 \dfrac{(\widetilde{S}_2 - a_2)^2}{2} + \alpha_2 g(x_2) \right]$

This is just the minimum period 2 cost of the inherited stock of extractions at the start of period 2, taking account of both damage costs and period 2 reduction costs, given the optimum choice of reduction in period 2.

The solution to this problem, the best choice for a_2 is:

$$\gamma_2 a_2 = \mathrm{E}[\ \delta_0 + \delta_1 (\ \tilde{S}_2 - x_2 - a_2) - \ \alpha_2\, g(x_2)\ ]$$

which is the just the condition that the marginal cost of alternative water equals the expected marginal benefit of use of alternative water in terms of the marginal reduction in damage costs.

We now consider the effect of variability. If we considered changes in σ^2 alone, keeping μ constant, then whilst the variance of x_2 will increase, so will the mean. So we express the expected value of *g* in terms of the mean of x_2 , = *k* say, and σ^2.

Letting $k = \mathrm{E}[x_t]$, then $\mathrm{E}[\ g(x_2)\ ] = \dfrac{\exp((1-\lambda)k - \lambda(1-\lambda)\sigma^2/2)}{(1-\lambda)}$ denote this by θ.

Hence, $a_2 = \dfrac{\delta_0 + \delta_1(\tilde{S}_2 - k) - \alpha_2\theta}{(\gamma_2 + \delta_1)}$.

It is clear that a_2 is increasing in both δ_0 , δ_1, and $\tilde{S}_2$, and falling in *k*. So that, as expected, the reduction in water use from the river in period 2 increases the more damaging extraction is to the wetland, and the greater extraction of water has been in the past. The effect of variability can be seen by considering increases in σ^2 for a constant k. This causes θ to fall, and so a_2 to increase, by an amount depending on λ which can be interpreted as the aversion towards risk.

So for high variance, high risk aversion, a_2 will be large. But there is a limit as to how large it can be. The effect of a_2 is to increase water levels in the Nura by using water from alternative sources. If all possible water flow along the Nura is subsumed into the base level of flow, then there is a constraint that $a_t \le b_t$. This is an irreversibility constraint which says that in each period it is not possible to increase water into the wetlands above what comes from the river[14]. Net extraction of water must be non-negative. This will affect not only what the period 2 decision is but also, because of the linkage between periods though the stock effect, what the period 1 decision should be.

If the availability of alternative water supply is sufficiently great in period 1, the irreversibility constraint will not bite in period 2, but if it is sufficiently low in period 1, then the irreversibility constraint will bite.

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[14] For one fifth of the years between 1935 and 1985 with four consecutive years 1936-39 flow was less than 5.44 m^3 / sec, whereas possible extraction could be up to 2.85 $m.^3$/sec. so this constraint is not implausible.

This will happen when $b_2 = \dfrac{\delta_0 + \delta_1(\widetilde{S}_2 - k) - \alpha_2\theta}{(\gamma_2 + \delta_1)}$, and as $\widetilde{S}_2$ depends on a_1 there will be a value $a_1^*(\sigma^2)$ for which the constraint bites if $a_1 < a_1^*$.

$a_1^* = (\delta_0 + \delta_1(b_1 - k) - \gamma_2 b_2 - \alpha_2\theta)/\delta_1$, a_1^* ↑as σ^2 ↑.

Period 1

In the first time period, we choose a value for use of water from the river, and consequently a_1 recognising its impact on the period 2 decision a_2, and consequent environmental damage.

So the planner's problem is:

$$\min_{a_1} \frac{\gamma_1 a_1^2}{2} + V(\widetilde{S}_2) = \min_{a_1} \frac{\gamma_1 a_1^2}{2} + V(\hat{S}_2 - a_1 - x_1) \text{ where } \hat{S}_2 = b_1 + b_2$$

for which, if $a_1 \geq a_1^*$, the solution for a_1 can be found in an analogous way.

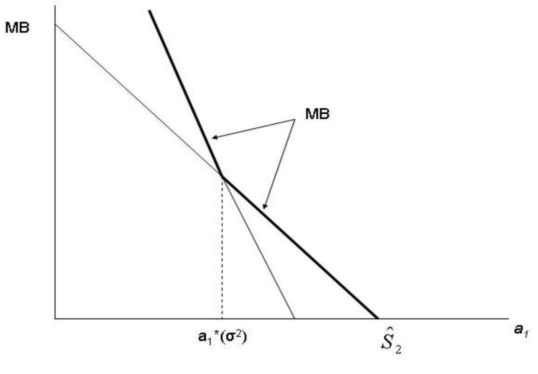
For $a_1 \leq a_1^*$ then we need to recognise the constraint that $a_2 = b_2$. The expected marginal benefit of a_1 (EMB_1) will be higher than it would be in the previous case, as shown in the diagram below, and is

$$E[\delta_0 + \delta_1(b_1 + b_2 - a_1 - b_2 - 2k) - \alpha_1 g(x_1)] = \delta_0 + \delta_1(b_1 - a_1 - 2k) - \alpha_1\theta$$

As σ^2 ↑ θ↓ and so EMB_1 ↑, and so a_1 ↑.

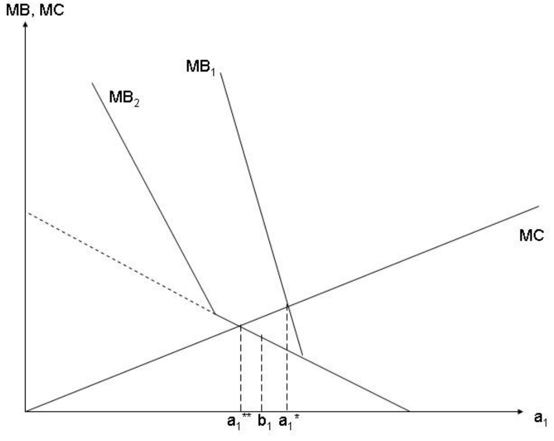
An interior solution for a_1 is given by $\gamma_1 a_1 = \delta_0 + \delta_1(b_1 - a_1 - 2k) - \alpha_1\theta$, or

$$a_1 = \frac{\delta_0 + \delta_1(b_1 - 2k) - \alpha_1\theta}{(\gamma_1 + \delta_1)}$$



Hence, irreversibility leads to at least as much use of water from alternative sources to the Nura as would be the case if there was no irreversibility constraint, and strictly more when the constraint bites. The effect of the parameters γ and σ are to increase the value of a_2 above the level it would be if we ignored the Log-Normal variability and the degree of risk aversion, and consequently it will increase the likelihood that the constraint $a_2 \le b_2$ bites i.e. $a_2 = b_2$, so that this will occur for lower values of δ than would otherwise be the case. So, first period water extraction will be lower for a wider range of the damage cost parameter than it otherwise might be. And hence, increasing variability increases the extent to which decisions are linked across time periods. It is possible that in the constrained case $a_1 > b_1$. In other words, the presence of the non-negative extraction constraint forces no extraction in period 1, and this can be triggered by that for period 2, which in turn depends on the amount of variability.

Hence, there will be parameter combinations, including a high degree of variability, where the constraint bites in both periods. In which case, the maximum alternative water use will be undertaken, and so no water would be taken from the River Nura at all. The possibility of this is shown in the diagram below. For these parameter values, including that of σ, the marginal benefit in period 1 arising from using water other than that from the Nura is MB_1. This kinks at a point to the right of the chosen level a_1*. So in this case, the irreversibility constraint in period 2 must bite, and so the marginal benefit of a_1 increases to allow for this. However, this causes the level of a_1 itself to be above the level of the period 1 constraint, and so both a_1 and a_2 are set at their maximum levels.



If the variance were lower, so that the marginal benefit of alternative water in period 1 were to be MB_2 , then the kink is to the left of the chosen level of a_1 in period 1, and $a_1^{**} < b_1$. In this situation, neither constraint bites, and water will be extracted from the river according to an unconstrained decision. Thus, increases in variance in water flow can lead to an optimal increase in precaution which precludes all water use.

This result is obtained for quadratic damage costs, so that there is no explicit catastrophe effect. If there were to be such an explicit effect, this would be an extra reason for being concerned about the irreversibility effect, as suggested by Narain and Fisher (1998). They argue that the analysis of irreversibilities needs to recognise the possibility of catastrophic effects, and in particular the possibility that such risks depend on the stock effects (in this case water level and salinity), so that reducing the effect on the stock reduces the risk of catastrophes – the risks are avoidable. They also argue that allowing for such effects would increase the significance of the environmental irreversibility.

Of course, if the uncertainty about flow, x, is revealed before current decisions about water use are taken then there will be a revision from the planned decisions based on an expected cost basis. For example, for period 2 the planned decision for a_2, a_2^P is given by

$$a_2^P = \frac{\delta_0 + \delta_1(\widetilde{S}_2 - k) - \alpha_2\theta}{(\gamma_2 + \delta_1)}$$

where $\widetilde{S}_2 \equiv (b_1 + b_2 - a_1 - x_1)$ and $\theta = \dfrac{\exp((1-\lambda)k - \lambda(1-\lambda)\sigma^2/2)}{(1-\lambda)}$

x_1 being the realised value for period 1 flow and k the expected value for period 2 flow.

The actual decision made, a_2^R after flow is known to be

$$a_2^R = \frac{\delta_0 + \delta_1(\widetilde{S}_2 - x_2^R) - \alpha_2(x_2^R)^{1-\lambda}/(1-\lambda)}{(\gamma_2 + \delta_1)}$$

There is a difference between a_2^R *and* a_2^P due to the realisation being different to the expected value. x_2^R replaces k, but there is an extra term in σ^2. A high realisation for x_2 leads to there being more water available for use, and this effect is enhanced by no longer taking variability into account.

As well as requiring a management plan that takes into account the effects of future uncertainty, if we know that this plan will come to an end, we also need a plan that will utilise any water surplus should such occur. However, if the plan is an on-going one, as Sustainability would require, then we will always want to take account of future shortages. This is a property of Time Consistency where we would never want to revise the plan because we do not learn anything that would make us want to change it. This arises in this model from the simple quadratic way in which almost all of it is expressed.

<u>Learning</u>

Learning will be important if it applies to parameters that enter in a non quadratic way, so leading to a non-linear response. Most noticeable of these is the damage cost parameter δ_1.

This can be seen from the result that $a_2 = \dfrac{\delta_0 + \delta_1(\widetilde{S}_2 - k) - \alpha_2\theta}{(\gamma_2 + \delta_1)}$.

δ_1 appears in both the numerator and denominator of this expression. By calculating the marginal benefit of period 1 action a_1, we can use analogous results to those of Ingham, Ma, Ulph (2005) to see what the consequences of including learning are. We focus here on uncertainty about δ_1 alone.

Suppose that the true value of δ_1 is unknown at the start of period 1, and may or may not be known at the start of period 2 depending on whether or not learning takes place. Compared to the case of no learning, or equivalently if a certainty equivalent procedure is used, by ignoring the irreversibility constraint because the value for a_1 is high, the prospect of future learning unambiguously leads to *lower* a_1. But, if there is an irreversibility constraint, then in general we cannot say whether the prospect of future learning should lead to a greater or lower value for a_1. However, if the irreversibility constraint bites whether there is no learning, or the marginal cost of a is sufficiently steep so that a_1 is low, then the prospect of future learning gives a *greater* value for a_1. So the impact of learning has no unambiguous effect on the amount of precaution.

Similar learning questions can be asked in relation to the clean up of mercury in the Nura. There is uncertainty related to how stable the mercury is in the river, and the possibility that it will reach the wetlands and lakes and cause damage to them. There is uncertainty related to the use of the water and its consequences once the mercury has been cleaned up, and the water is available for use. There is also uncertainty related to the stability of the mercury in the technogenic silt, and whether this is moving along the river or is stationary. Suppose the decision concerns how much of the mercury silt to remove, based on the damage cost of leaving the silt where it is. This could be low because the silt stays where it currently is, close to the site of its original deposition, or high, because mercury can change form, and travel down the river. Irreversibility arises because, once the mercury silt has travelled down the river, it is harder to remove. For this problem the analysis of the previous section would apply. However, a rather different structure would apply to a decision which is either to fully clean up or not clean up.

5. Robust Policies

This section discusses policies which have been recently proposed in several areas where large uncertainties exist. When these apply to the decision model itself, they make conventional approaches unsatisfactory, and suggest the need for ‘Robust’ or ‘Safe’ policies. For water problems, policies that are described as ‘Robust’ have been proposed by Roseta-Palma and Xepapadeas, (2004). For climate change problems, there is a large literature, for example Lempert and Schlesinger (2000), and for Macroeconomic and Monetary Control problems, Hansen et al (2001) is an example.

These studies are all concerned with what has been described as 'deep' or 'model' uncertainty. The question is what the level of the decision variables should be when the model being used to determine those decisions is not known. Hansen et al. (2001) show how robust control theory justifies the use of a decision criterion which is that of max-min expected utility, where the maximization is done with respect to the decision variables, and the minimization with respect to uncertain variables in the control model. They consider several possibilities for what robustness might mean. These are:

1. Risk Sensitive Control Problem. This introduces a risk sensitivity parameter into the objective function which allows for an enhanced response to risk. In this form of robustness, there is no concern about a mispecified dynamic structure. Instead the objective has an extra enhancement due to risk that is captured by the variance of the Value function for the problem.

2. Multiplier Robust Control Problem. This imposes a positive penalty on deviations of the true model away from a single approximating model, so that we wish to take into account in the overall value of the objective function costs arising from use of the wrong model

3. Constraint Robust Control Problem. This imposes a constraint on the maximization problem derived from the uncertainty in model specification. By including an additional state and control variable to take account of this, the model generates a recursive structure in which history matters, so that a once and for all initial decision cannot be made. This version of the problem links up directly with that of a min-max expected utility.

All of these turn out to be equivalent. So, where a decision maker is unsure about what the appropriate model is, a max-min approach is appropriate, in the sense that it leads to the same decision rules, as the other rather different structures leading to model uncertainty. Min-max expected utility is interpreted as being aversion to uncertainty, and these equivalent approaches are ways in which that aversion might be implemented.

Roseta-Palma and Xepapadeas (2004) consider a single time period model and the management of ground water under uncertainty. Uncertainty applies to the distribution function for the stochastic process determining water supplies. Essentially, there is uncertainty about what the appropriate model for water supplies is, and robustness concerns a policy taking that into account. They use a decision rule which is a max-min expected value, where the minimisation is with respect to the model uncertainty. The objective is to choose the optimal value for the control for the worst possible outcome for what is the model. Take a single time period model, in which the model uncertainty is additive to the flow of water. For a quadratic objective function, an expression can be calculated for water use which depends on a term given by the ratio of the marginal cost of water use to the penalty parameter which comes from how sure we are about the model being used. This term increases with uncertainty about the model.

As an example, take the two period decision problem for water use as set out in the previous section and derive a robust policy for period 2.[15] Using the decision problem of water allocation, and the previous Log-Normal distribution for water flow in the river, we can see how a robust policy might be developed. The decision problem for the allocation of water in a two period model as set out above is :

$$V(\widetilde{S}_2) \equiv \min_{a_2} E\left[\frac{\gamma_2 a_2^2}{2} + \delta_0 (\widetilde{S}_2 - a_2 - x_2) + \delta_1 \frac{(\widetilde{S}_2 - a_2 - x_2)^2}{2} + \alpha_2 g(x_2)\right]$$

To see how the robust policy is obtained we first consider robustness to mis-specification of $\widetilde{S}_2$. As this enters the objective in a quadratic way, it is straightforward to derive such a policy. Following the max min expected value procedure towards robustness the problem becomes

$$\max_{a_2} \min_{h} \left(-\frac{\gamma_2 a_2^2}{2} - \delta_0 (\widetilde{S}_2 + \varepsilon + h - a_2 - x_2) - \delta_1 \frac{(\widetilde{S}_2 - a_2 - x_2)^2}{2} - \alpha_2 a_2 g(x_2) + \varphi h^2 \right)$$

where φ represents the penalty to be attached to mis-specification. It represents how sure we are that the true mean for $\widetilde{S}_2$ is in fact $\overline{S}_2$.

This gives first order conditions assuming an interior solution of

a_2 : $0 = -\gamma_2 a_2 + \delta_0 + \delta_1 (\widetilde{S}_2 + h - a_2 - k) - \alpha_2\theta$

h : $0 = -\delta_1(\widetilde{S}_2 + h) + 2\varphi h$

These have a solution $h^* = \delta_1 \widetilde{S}_2 / (\delta_1 + 2\varphi)$ and $a_2^* = \dfrac{\delta_0 + \delta_1 (\widetilde{S}_2 \left(\dfrac{2\delta_1 + 2\varphi}{\delta_1 + 2\varphi}\right) - k) - \alpha_2 \theta}{(\gamma_2 + \delta_1)}$

The robust strategy is to add an amount h^* on to $\widetilde{S}_2$ as a safety or robustness premium against mis-specification of what $\widetilde{S}_2$ is and then calculate a_2 based on that. In this case taking robustness into account is a simple additive procedure. Robustness concerning the distribution of water flow which is a more complex problem.

Suppose now that it is the water flow in period 2, x_2 that we may mis-specify and so want to develop a robust policy against. We use a model for x_2 : $k = k_2 + \varepsilon + h$ where ε has zero mean and h affects the mean of x_2 only.

Following the max-min expected value procedure for robustness the problem becomes:

[15] It is possible to derive analogous results for period 1.

$$\max_{a_2}\min_{h}\left(-\frac{\gamma_2 a_2^2}{2}-\delta_0(\widetilde{S}_2-a_2-k-\varepsilon-h)-\delta_1\frac{(\widetilde{S}_2-a_2-k-\varepsilon-h)^2}{2}-\alpha_2 a_2 g(x_2+\varepsilon+h)+\varphi h^2\right)$$

φ now represents how sure we are that the true mean for x_2 is in fact k.

This gives first order conditions assuming an interior solution of

FOC: a_2 $\quad \left(-\gamma_2 a_2+\delta_0+\delta_1(\widetilde{S}_2-k-h-a_2)-\delta\alpha_2 g(k+h)\right)=0$

FOC: h $\quad \left(\delta_0+\delta_1(\widetilde{S}_2-k-h-a_2)-a_2 g'(k+h)+2\varphi h\right)=0$

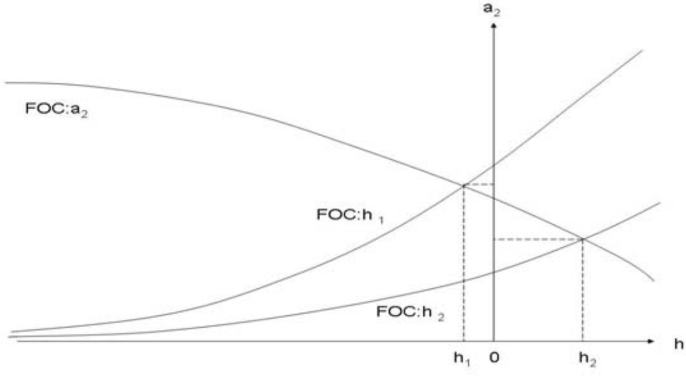
Taking the expectation with respect to the true Log-Normal distribution, standard results give the pair of simultaneous equations:

$a_2 = \dfrac{\delta_0+\delta_1(\widetilde{S}_2-k-h)-\alpha_2\theta(h)}{(\gamma_2+\delta_1)}$ where $\theta(h)=\dfrac{\exp((1-\lambda)(k+h)-\lambda(1-\lambda)\sigma^2/2}{(1-\lambda)}$; and

$a_2 = \dfrac{\delta_0+\delta_1(\widetilde{S}_2-k-h)+2\varphi h}{(\gamma_2+\alpha_2\theta_1(h))}$ where $\theta_1(h)=\exp(-\lambda(k+h)+\lambda(1+\lambda)\sigma^2/2)$

Whilst an analytic solution cannot be obtained for these equations, we can plot the two equations giving the first order necessary condition[16]:

When $2\varphi>\delta_1$, the necessary condition for h and a_2 are illustrated below:



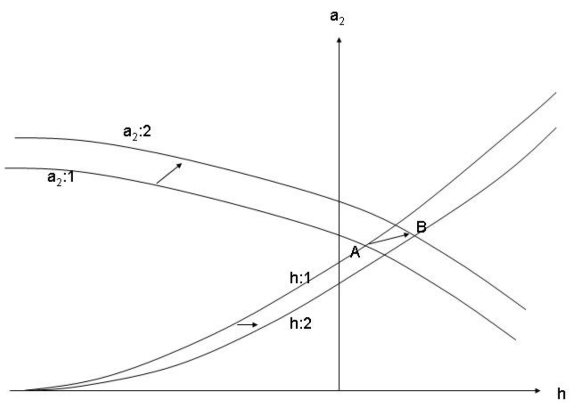
Whether h is positive or negative[17] depends upon whether the stock effect of extra water in the wetland $\widetilde{S}_2$ dominates the effect on the action undertaken to mitigate damage, a_2. For the stock effect, extra water flow into the wetlands is good news as it reduces damage, so the

[16] Unfortunately, it is not possible to obtain an explicit solution for robustness to the distribution of river flow. A comparative static exercise allows for the effect of various parameters on the outcome.
[17] h positive implies that the 'worst' true outcome for the mean of flow is above that which would be used in determining the plan where we are fully sure of the model.

worst outcome is when actual flow is below that planned for. For the effect that comes through the interaction term between x_2 and a_2 , extra flow is bad news as it means that the level of a_2 chosen to reduce damages was higher than it need have been so that some of the cost of this action could have been avoided if the mis-specification of x_2 had not been made.

By seeing how the two curves shift with respect to the parameters γ, μ, σ, θ, δ we can see how the period 2 water policy changes with these parameters. It also shows how they interact with each other, although this will depend on the model mis-specification that is introduced.

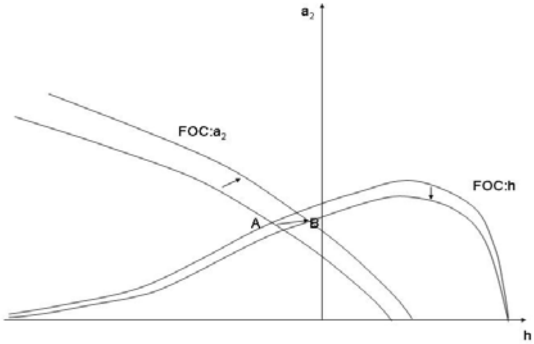
The diagram below shows how the two curves move as variability increases in the case that $2\varphi > \delta_1$. In this case the FOC: h curves slope upwards, and have the same abscissa. Increases in variability reduce the slope. Whilst for the FOC: a_2 curve, an increase in variability shifts the curve upwards.



Quite what happens depends on parameter values and whether the shift in the FOC:a_2 curve dominates or not the swivel in the h curve. This reflects the opposing forces of variability on the robustness with respect to level of water in the wetlands, and the costs of alternative actions.

For $\delta_1 > 2\varphi$, the diagram looks as follows. The FOC:a_2 curve is unchanged but the FOC:h curve has negative slope, and a positive abscissa. It can be shown that the abscissa for the FOC:h curve is greater than that for the FOC:a_2 curve and this implies that again there is an ambiguous effect on a_2 as variability increases.

In the case where a_2 increases as variability increase then, as before, there will be an upper constraint on the value that a_2 can take.



A more general use of the interpretation of the max-min version of robustness as making the best choice in the worst possible scenario is as follows. These strategies are equivalent to the 'safe' strategies that have been proposed for climate change. A safe strategy would be to avoid irreversible damage to the wetland and lake system. If the state of the wetland complex is determined by a differential equation, this would mean keeping the system within the reversible part of the solution. Given the current state of knowledge of the wetland, we cannot be entirely sure where the boundary value of *b* and range for that corresponds to a high ecological state lies, so that we are in the potential catastrophe case of Tsur and Zemel (1998) as discussed in section 3. In this situation the optimal policy was to use past experience to determine a safe region which avoided catastrophic effects. An added complication is that we need to take into account the effect of the randomness of water flow. Standing still may not be a desirable outcome.

However, suppose we are able to compute, for the current water levels, what is the worst outcome for water level, over the longest experienced dry period if that started now, for a given confidence interval. We can compare this with the worst outcome that there has been for an acceptable ecological status, and what current water level would give rise to that. This gives a current water surplus that can be safely used and diverted away from the wetlands. By calculating this surplus for each period, a safe plan for water use can be determined, and an expected value for water use calculated. This is the equivalent here of the safe corridor proposed for Greenhouse Gas emissions[18]. Alternatively, it is equivalent to a safe policy for a pension fund, with future commitments, which give a minimum value constraint but with uncertain payouts and returns. Such a problem has been studied by Gerber and Shiu (2003), for example. In this context, the lake and wetland complex can be viewed as a buffer stock, in which the water ,not used but allowed to flow into the wetland, corresponds to precautionary savings[19]

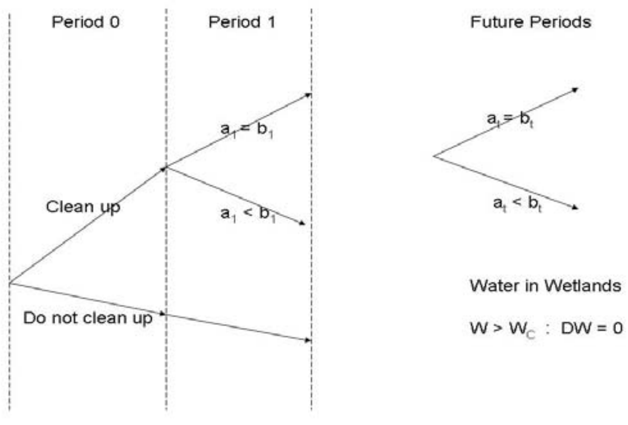
[18] see Bruckner et al. (2003)
[19] see for example Aizenmann (1998).

6 Conclusions

Consequences for the Mercury Clean Up Decision

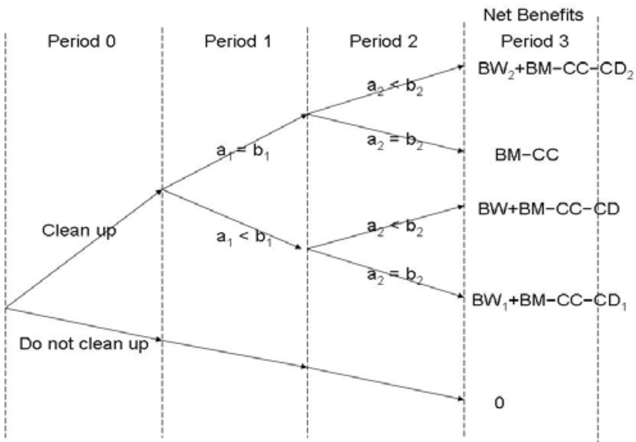
We contrast here two extreme versions of the appraisal of the Nura Mercury Clean Up Project, based on the issues concerning variability that we have discussed. In the first case, there is little difference between mean and median flow, and flows in different years are independent of each other. These correspond to the variability of water flow being low. This is the situation that would correspond to a western European river. Here, the flow corresponds to that arising from an Arithmetic Brownian Motion. As the expected hitting time for a lower boundary to be reached is not finite, it may be concluded that damage to the wetland will be at a low level. The water level remains above that at which substantial damage occurs. Loss of water level in one period can be made up in future periods, so that the period 1 decision is replicated in future time periods. So whilst extraction may be zero in very low flow years, there is no consequence for previous years' decisions.

We represent this decision process by the following decision tree. The end decision compares the clean up cost, CC with the benefit from mercury removal BM, the benefit from water use over future periods, BW, but as damage to wetlands doesnot occur, DW = 0.



The second extreme is one in which there is very high variability of flow where the mean is much higher than the median, and the water level in the river and the wetlands correspond to a Geometric Brownian Motion. This generates a correlation between water levels in successive years and periods of wet and dry periods, and so is a stylised version of rivers and wetland systems such as that found on the Nura.

In this case, there is a finite expected time at which the critical value will be reached. This depends on the amount of water removed, and the variance of the process. When this critical value is reached, there is the possibility of high damages to the wetlands, so we need a decision structure where these damages can be taken into account. This is shown in the decision tree based on the example we used in section 3. The decision taken in period 0 is now based on the net benefits realised in period 3. The various levels of net benefits are shown depending on the extraction decisions taken. In period 2, the decision taken is to maximise the value of the objective (or equivalently minimise cost) based on decisions taken up to that time, and the realisation of stochastic flow into the wetlands. High variability, together with a desire for making a robust decision against parameter mistakes, can lead to the decision not to use water at all. This depends on, and interacts with, period 1 decisions. So that, as in our example, it may be desirable not to use water in period 1, because of the consequences for the period 2 choice, and ultimate damage to wetland. The clean up decision at period 0 will depend on the expected value of net benefits for the four options compared to with 0, the base level value for 'doing nothing' .



The main difference compared to the previous decision tree is that of increasing the chances of making zero water use for each period, because (i) we take into account damage to wetland (ii) we recognise impact of variability on water use decision and (iii) we recognise the interaction between decisions in adjacent periods caused by the irreversibility effect.

We can extend the decision tree to planning over many, rather than just three, periods. Suppose that damage to the wetlands is realised when the water level in the wetlands, or river flow, reaches a critically low level, and consequently the damage level reaches a high level.[20] The formal solution to this by the backward induction method is straightforward in principle,

[20] This could be indicative of forthcoming catastrophe

although an explicit expression for water use in each period is exceedingly cumbersome. Depending on the random flow of water in the river and into the wetlands, there will be the possibility of irreversibilities where, if possible, it would be desirable to have negative water extraction in period t, but the non-negativity constraint forces extraction to be zero. This increases the benefits of water conservation in the previous period, $t - 1$. Our example showed that it is possible that this increase in benefit, which depends in part on the river flow variability term, is sufficient to cause water extraction in period $t - 1$ to be negative, if that were possible. The non negativity constraint then has a similar effect on period $t - 2$, and so on back to the start when the clean up decision is being made. Considering the value of water flowing into the wetland, when the critical damage level is reached, has an effect that 'cascades' backwards, with the result that no extraction of water is ever undertaken. Hence, if the clean up decision hinges on the value of extracted water, clean up would no longer be economically viable.

If we neglect to take into account the variability in river flow, or we use the same decision tools as for rivers with normally distributed flow, then when the clean up decision is made, there is the possibility of overlooking the presence of the threshold level. Hence, ecological damage to the wetland is not included, and, in any case, because the variability term does not enter into the planned extraction levels, the irreversibility effects may not be triggered. Consequently, water would then be used in all periods and this could justify the clean up decision.

Overall Conclusions

All rivers have variable flow. The River Nura has very different flow characteristics to rivers in Western Europe. The Log-Normal distribution has been found to be a good representation for river flow. It is even more useful here as a way of representing the high variability of flow and the correlation of flows over years in which there have been long runs of successive high and low flow.

Moving away from Normal Distributions for Uncertainty and Quadratic Objective functions leads to many difficulties for tractability. So it is not too surprising that, in practice, uncertain variables are often replaced by expected values. However, techniques do exist and they have been widely applied in the area of financial economics. Such techniques have much promise for use in these areas. We have only used one such technique which is to contrast when different stochastic processes, that model water flow in a river, will reach a boundary value at which irreversible or catastrophic ecological damage occurs. By contrasting that for a Normal and Log-Normal we can see that whilst a Normally distributed process for flow has infinite expected time of reaching such a boundary, for a Log-Normally distributed process, there may be a finite expected time of reaching this boundary depending on the variance.

For the Log-Normal distribution, it is possible to calculate the expected value for where part of the value for the objective satisfies a Constant Relative Risk Aversion property. Using this, we were able to show how increasing variance and aversion to risk will increase the amount of water that would be obtained from alternative sources to the Nura. This then goes beyond just advocating a precautionary approach by calculating the amount of precaution needed.

Two important conclusions relate to the types of management structures that might be applicable for the River Nura, and the Kurgald'zhino wetlands. The first is that we used a decision framework corresponding to one of Weak Sustainability. That is, we used an expected Net Benefit approach in which we allowed for trade offs between environmental costs and the costs of taking action to prevent environmental damage. But we ended with a situation where the optimal decision, that of not allowing for any extraction of water, could correspond to one of Strong Sustainability in which there is no trade-off. The presence of irreversibilities and variability could generate a sufficient amount of precautionary behaviour that no water use would be allowed. Damage would be strictly minimised in that it would be limited to that arising from the variability in natural water flow.

The second conclusion relates to this, but concerns how a management plan should be implemented. A usual contrast is made now in the environmental economics literature between Command and Control (CAC) methods and Market Based Approaches (MBA). For many reasons, MBA is often recommended. The model here, however, presents a situation in which it would be difficult to obtain the right outcome through MBA. In the case where variability and potential damage to the wetlands requires no water use, it would require there to be sufficient bids for the wetlands to be put in to limit water use. In the extreme, these bids would need to be sufficiently large that all other users are bought out. Thinking of the store of water in the wetlands as being an asset, such as a pension fund, where not using water is analogous to precautionary saving, this suggests that something analogous to financial futures markets would be needed to ensure that future consequences related to uncertainty have a presence in current decisions. However, the outcome where no water is ever used would suggest that such a market is likely to be thin and not operate efficiently, if at all. So that a CAC method, such as management by a River Basin Authority, may well be a more satisfactory structure. Of course, this just reflects the well known correspondence of Weak Sustainability with MBA and Strong Sustainability with CAC.

We finally conclude that in transferring experience from one part of the world to another, such as is implicit in projects under the Water Framework Directive of the European Union, and World Bank projects, the assumptions underlying the treatment of uncertainty should be very carefully considered. It is necessary to start from the characteristics of the river basin and then build a decision framework that embodies those characteristics, rather than imposing general decision making tools, such as Certainty Equivalent Cost Benefit, since their basic assumptions mayl not be satisfied.

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